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Innovative Projects Scheduling with Scenario-Based Decision Project Graphs

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Abstract. Decision project graphs – DPG are treated as a combination of deterministic and stochastic networks. They are designed for projects where at some stages at least one activity from a set of alternative activities is supposed to be performed. A given alternative task may differ from other activities belonging to this set in respect of times, costs and even sets of successors. Decision project graphs are used in project planning, scheduling and management. Due to the fact that especially in the case of innovative projects many factors are not completely known before the project execution, the DPG issue has been already investigated both under certainty and uncertainty. In this contribution we present a novel scenario-based DPG rule which takes into consideration possible scenarios, dependent activity durations, the decision-maker's attitude towards risk and the distribution of parameter values connected with particular activities. The procedure is especially designed for totally new (innovative) projects where it is complicated to estimate probabilities of particular scenarios since no historical data are available. The decision rule is assisted with an optimization model which can be easily solved with the use of diverse optimization computer tools. The model may support both reactive and proactive project management.

Keywords: decision project graphs, innovative projects, time-cost project management, optimization, uncertainty, risk, scenarios.

JEL Classification: C44, C61, D81, O22

Conference topic: Contemporary Financial Management.

Introduction

Decision project graphs (DGP) were proposed by (Crowston, Thompson 1967) and other researchers (Hastings, Mello 1978) in the sixties and seventies of the 20th century. Decision project graphs, called also decision critical path method networks (DCPMN), are used in project planning, scheduling and management. Originally, DGPs referred to the concept of multiple choices at alternative nodes when decision-making was of deterministic nature (San Cristobal 2015). Nevertheless, in connection with the existence of many uncertain parameters describing the project (times, costs, resources, network structure), scientists started to investigate DGPs under uncertainty (Thompson 1968; Pollack-Johnson, Liberatore 2005). In their research it is assumed that the probabilities of the occurrence of particular scenarios may be estimated. In this contribution we would like to analyze the aforementioned topic in the context of totally new projects (new product development project, process development project, technology-implementation project, research project or pharmaceutical development project), where the likelihood (understood as frequency) cannot be known since no sufficient historical data are available. In such circumstances probability-like quantities may be applied. The novel scenario-based DPG rule presented in the paper takes into consideration possible scenarios, the decision-maker's attitude towards risk (measured by the coefficient of optimism) and the distribution of parameter values (asymmetry) connected with particular jobs. The procedure may support both reactive and proactive project management.

The paper is organized as follows. Section 2 deals with the main features of the traditionally understood decision project graphs. Section 3 describes DGPs under uncertainty. Section 4 presents a scenario-based DPG rule for innovative projects. Section 5 provides an illustrative example. Conclusions are gathered in the last part.

Decision project graphs – description

Decision project graphs are a generalization of Critical Path Method networks. They are treated as a combination of deterministic (DAN, Deterministic Analysis Network) and stochastic (GAN, Generalized Analysis Network) networks: deterministic since they do not contain cycles; stochastic since only a part of activities (tasks, jobs) considered in the graph are finally performed (with a non-negative probability). Deterministic graphs are suitable for simple projects with an explicitly defined technology (Spinner 1981), meanwhile stochastic graphs (Neumann, Steinhardt 1979) are appropriate for innovative projects where changes are possible during the project realization.

Decision project graphs are designed for projects where at some stages only one or at least one activity from a set of alternative activities (tasks) is supposed to be executed. A given alternative task may differ from other activities belonging to the set aforementioned in respect of times, costs and even sets of successors. Note that if we decide to do one of the alternative tasks, then all immediate precedence relations that the activity satisfies must hold in the final graph. On the other hand, if we decide not to perform a given alternative task, then none of its immediate precedence relations hold and we ought to remove that activity that impinge on it from the decision project graph (San Cristobal 2015).

DPGs may be presented by means of AON Activities on nodes) or AOA (Activities on arcs) techniques. The first technique uses nodes for activities (tasks, jobs) and arcs for precedence relations. The second one represents tasks with the aid of arcs and events on the basis of nodes. Here, we are going to apply the second graphical approach (i.e. AOA).

Now, let us define in detail the structure of the decision critical path network. We assume that each DPG contains a set of job sets $S = \{S_1, S_2, \ldots, \ldots, S_i, \ldots, S_{n-1}\}$, where *n* denotes the number of nodes in the graph. Some job sets are *deterministic job sets* (each task must be done), others can be named *alternative job sets* since they have several alternative elements (one or at least one task has to be done): $S_i = \{A_{i1}, A_{i2}, \ldots, A_{ik(i)}\}$. If all job sets are deterministic, they are related to deterministic nodes and the network can be reduced to a deterministic graph. Once decisions are made for each set of alternative activities (represented by an alternative node), the DPG may be also converted into an ordinary deterministic graph. The DPG contain *n* nodes, being simultaneously events, and $\sum_{i=1}^{n-1} k(i)$ activities.

It is worth emphasizing that when applying the AOA technique, graphs often contain so-called dummy activities which are represented by dotted arrows (Moder, Phillips 1964). Their time is equal to zero. They are not related to the realization of any real task. Dummy activities are just used to guarantee each necessary precedence relation in the graph and to include time and technological interdependencies between jobs.

Hence, the set of nodes can be divided into three subsets:

- DET(N) – deterministic nodes (connected with deterministic job sets);
- ALT(N) – alternative nodes (connected with alternative job sets);
- END(N) – end nodes (with predecessors and no successors) – it is desirable to have one end node (target) in the graph.

The set of activities can be divided into the following subsets:

- DET(A) – deterministic tasks belonging to deterministic job sets and starting from deterministic nodes;
- ALT(A) – alternative tasks belonging to alternative job sets and starting form alternative nodes;
- DUM(A) – dummy activities.

Theoretically, it is possible to insert mixed nodes (i.e. being simultaneously the beginning of deterministic and alternative asks) in the graph.

The AOA-based optimization model allowing the decision maker (DM) to minimize the project duration and to select suitable alternative activities may be as follows:

$$t_n \to \min \tag{1}$$

$$t_1 = 0 \tag{2}$$

$$t_j \geq t_i + t_{ij} \qquad <i,j> \in DET(A) \tag{3}$$

$$t_j \geq (t_i + t_{ij}) \cdot x_{ij} \qquad <i,j> \in ALT(A) \tag{4}$$

$$\sum_{<i,j> \in S_i} x_{ij} = 1 \qquad i \in ALT(N) \tag{5}$$

$$t_j \geq t_i \qquad <i,j> \in DUM(A) \tag{6}$$

$$t_i, t_j \geq 0 \qquad i,j \in ALT(N), DET(N), END(N) \tag{7}$$

$$x_{ij} \in \{0,1\} \qquad < i, j > \in ALT(A) \tag{8}$$

where: n – number of nodes in the graph (parameter); t_n – completion time of the whole project (continuous variable); t_i, t_j – times of events i and j (continuous variables); t_{ij} – duration of activity $<i,j>$ (parameter); x_{ij} – binary variable connected with alternative activity $<i,j>$ (it is equal to 1, when this task is done; otherwise it is equal to 0); S_i – job set i containing alternative activities starting from node i (parameter).

Note that the optimal value of variable t_i belongs always to interval $[t^I_i, t^{II}_i]$, where t^I_i denotes the earliest time of event i (the earliest time at which node i can be reached such that all its preceding activities have been finished) and t^{II}_i signifies the latest time of event i (the latest time that node i can be left such that it is still possible to finish the overall project within the minimum completion time t_n), see (Anholcer, Gaspars-Wieloch 2011, 2013; Gaspars-Wieloch 2012).

As was mentioned before, sometimes it is required to perform not one, but at least one activity from a given job set. If exactly one of the tasks must be executed, then the mutually exclusive interdependence condition is expressed by Eqn (5). Depending on the decision-maker's preferences, that formula may be replaced by Eqns (9), (10) or even (11).

$$\sum_{<i,j>\in S_i} x_{ij} = b_i \qquad i \in ALT(N) \tag{9}$$

$$a_i \le \sum_{<i,j>\in S_i} x_{ij} \le b_i \qquad i \in ALT(N) \tag{10}$$

$$c_{il} \le \sum_{<i,j>\in S_i} x_{ij} + \sum_{<l,m>\in S_l} x_{lm} \le d_{il} \qquad i,l \in ALT(N) \tag{11}$$

where: b_i – exact or maximal number of alternative activities belonging to job set i that should be done (parameter); a_i – minimal number of alternative activities belonging to job set i that should be done (parameter); c_{il}, d_{il} – minimal and maximal total number of alternative activities belonging to job sets i and l that should be done (parameter).

The model (1)–(8) has been formulated by the author and its target consists in minimizing the project completion time. Other optimization models for DPGs can be found in the literature. The Decision Critical Path Method (DCPM) can be applied to problems having a discrete time-cost tradeoff. For instance, (San Cristobal 2015) following (Crowston, Thompson 1967; Crowston 1970) presents a model minimizing the project completion cost with a desired project completion time. That model includes both the activity performance costs and a penalty (or reward) if the project is completed after (or before) the due date. A similar model was proposed by (Hindelang, Muth 1979), but this time authors refer to dynamic programming. (Grudzewski 1985) formulates a model minimizing direct project costs subjected to a deadline constraint, see the so-called deadline problem (Anholcer, Gaspars-Wieloch 2013, 2011).

Decision project graphs with uncertain parameters

We assumed in the previous Section that all parameters were deterministic (i.e. exactly known). However, especially before the realization of innovative projects, such characteristics as durations, costs, resources may be unknown or known not completely. The ability to schedule and plan uncertain projects is regarded as an extremely vital skill in project risk management which constitutes one of the nine Knowledge Areas within the Project Management Institute's *A Guide to the Project Management Body of Knowledge* (*PMBOK® Guide*), PMI 2000. The importance of addressing uncertainty related to activity duration and other project parameters can be seen by considering the results of surveys regarding available project management software and practitioner use of project network analysis methods (Pollack-Johnson, Liberatore 2005).

It is worth mentioning that the term "uncertainty" and "risk" are interpreted in different ways in the literature depending on which theory is recognized: theory of decision or theory of economics (Birge, Louveaux 2011; Dubois, Prade 2012; Gaspars-Wieloch 2015c, 2016a, 2016b, 2017; Guney, Newell 2015; Kaplan, Barish 1967; Kmietowicz, Pearman 1984; Knight 1921; Magruk 2016; Merigo 2015; Trzaskalik 2008; von Neumann, Morgenstern 1944; Voronova 2008; Ward, Chapman 2003). Here, we assume that uncertainty involves all situations with non-deterministic parameters (known or unknown probability distribution, lack of information about possible events), while risk is related to the possibility that some bad (or other than predicted) circumstances will happen. We define uncertainty as a situation where alternatives may lead to different effects and the probability of scenarios is known or not, but if not, some probability-like quantities may be often estimated and applied.

In the case of uncertain parameters, the description of the project may be given by means of interval values, scenarios, probabilities (Azeem *et al.* 2014), probability-like quantities, fuzzy numbers (Błaszczyk *et al.* 2013; Okmen, Oztas 2014), etc. In decision project graphs uncertain values may concern both deterministic and alternative tasks.

Let us first briefly discuss some project scheduling uncertainty procedures already worked out and presented in the literature. Note that they may be applied to different kinds of network, not only to DPGs. The most well-known technique is Program Evaluation and Review Technique – PERT (Malcolm *et al.* 1959) which considers uncertainty related to activity duration and collects optimistic, the most likely and pessimistic duration estimates for all tasks (three-point estimation technique). PERT may be used to find the expected length of the critical path (a critical path is a path from the beginning of the network to its end, consisting only of activities with a total slack equal to 0; any delay in those activities entails a delay of the whole project). The criticality of particular activities and paths as well as the probability distribution of the project duration can be also estimated on the basis of Monte Carlo simulation which ensures a highest probability of completion and is used in extremely many domains, e.g. (Suhobokov 2007; Zhang *et al.* 2015). Monte Carlo simulation generates random values for inputs that are processed through a mathematical model in order to generate multiple scenarios. The distribution type (normal, exponential, uniform, etc.) is specified by the user.

Besides methods connecting uncertainty with times, costs or resources (i.e. activity characteristics), there are also approaches which treat the whole project structure (i.e. network logic) as uncertain. In stochastic networks (GAN) only a part of jobs (with a non-negative probability) considered in the graph are finally performed. These graphs enable freely selecting activities during the project execution (Gaspars-Wieloch 2008). Among stochastic networks it is worth mentioning GERT (ang. Graphical Evaluation and Review Technique) and GERTS (ang. Graphical Evaluation and Review Technique Simulation) which allow a probabilistic treatment of both network logic and estimation of activity duration (Pritsker 1966, 1979; Wiest, Levy 1977). Those procedures assume that repeated activities within loops are possible.

The main drawbacks of methods enumerated above are as follows. Firstly, in all methods independence of duration distributions is assumed (i.e. the time of a given task does not influence the time of another activity). Secondly, the choice of the Beta-distribution in PERT was intuitive – nevertheless, some practitioners defend that choice since it has a satisfying smooth shape and it can represent skew (Broadleaf 2014). Thirdly, PERT concentrates on a single path through the network and takes no account of the possibility that parallel paths can become critical. Fourthly, in GERT all repeated tasks within loops have identical duration distributions (Neumann 1990). In connection with the disadvantages aforementioned, researchers and practitioners are trying to modify and improve existing methods, e.g. (Dodin 2006).

(Pollack-Johnson, Liberatore 2005) stress that a micro-oriented uncertainty modelling merely at the activity level may be insufficient since there are situations where the uncertainty is related to different sequences or combinations of events occurring or not occurring. Therefore, they suggest relating uncertainty to different project scenarios rather than individual tasks. Those authors propose a scenario macro-level approach for modelling and analyzing projects with significant uncertainty in their network structure or durations of some jobs. The procedure combines critical path analysis (i.e. a method determining the critical path and the minimum project completion time by performing forward and backward passes through the project network) and probability analysis with Microsoft Project and Excel. (Pollack-Johnson, Liberatore 2005) express the uncertain network structure through a set of network scenarios*,* each having a specified probability of occurrence. They apply AON technique – arrows for precedence relations and dotted arrows to indicate uncertain precedence relations. Their approach allows calculating expected criticalities and total slacks of each activity. The method refers to conditional project planning. That procedure gives the possibility to analyze projects with dependent activity durations and is applicable for contingency planning. Authors of the described approach underline that it uses software and techniques familiar to (and easily accessible by) almost all project managers.

There are also researchers investigating another type of uncertainty in project scheduling, i.e. activities which may fail during the project execution (De Reyck, Leus 2008). (Creemers *et al.* 2013) present a method considering technological and duration uncertainty. It is based on stochastic dynamic programming and incorporates both the risk of activity failure and the possible pursuit of alternative technologies.

Additionally, we may find in the literature papers devoted to stochastic project networks optimized in respect of the net present value (Benati 2006; Creemers *et al.* 2010; Sobel *et al.* 2009) or under resource constraints (Igelmund, Radermacher 1983).

As was mentioned before, ideas presented above concern almost all types of networks, not only DGPs. Now, we would like to focus on decision project graphs under uncertainty. One of possible ways to insert uncertain factors in DPGs is the model called CAAN (ang. Controlled Alternative Activity Network) proposed by (Golenko-Ginzburg 1988). The author assumes that the project has two different types of alternative events. The first one reflects stochastic (uncontrolled) branching of the development of a project. The second one is of a deterministic nature, i.e. the project decision-maker chooses the outcome direction. The problem of controlling a project is the choosing of an optimal outcome direction at every 'decision-making' node which is reached in the course of the project execution. This is carried out by a permanent reduction of the initial network and by applying lexicographical scanning. On the other hand, GAAN (ang. Generalized Alternative Activity Network Model) based on the lexicographical method and discrete optimization, and developed by (Golenko-Ginzburg, Blokh 1997) takes into account deterministic tasks, alternative stochastic tasks and alternative deterministic tasks. The procedure enables analyzing different kinds of project managers (risk-neutral, risk-seeking, risk-aversion decision makers). (Voropayev *et al.* 2013) suggest CCANM (ang. Controlled Cyclic Alternative Network Model) which unifies two formerly developed network models: CANN and the cyclic GERT (with loops and different logical relations).

Scenario-based DPG rule for innovative projects (SB-DPG rule)

In the previous Section we had the opportunity to become acquainted with existing methods applied to project optimization when some parameters and factors concerning particular activities and the project as a whole are uncertain. Note that those procedures may be useful when we accept the type of probability distribution adopted in a given approach or we know exactly the probability of occurrence of each scenario! However, in the case of innovative projects the objective (or even subjective) probability may be unknown (or difficult to estimate) due to: 1) the lack of historical data about already executed similar or identical projects, 2) the lack of sufficient knowledge about the mechanism, circumstances, conditions, required resources, possible obstacles, 3) the one-shot character of the project (see one-shot decisions defined in (Guo 2011)) and the fact that for a single event the mathematical probability understood as frequency cannot be computed (von Mises 1949). That is why, we would like to present a new approach which does not require any information about the likelihood of scenarios.

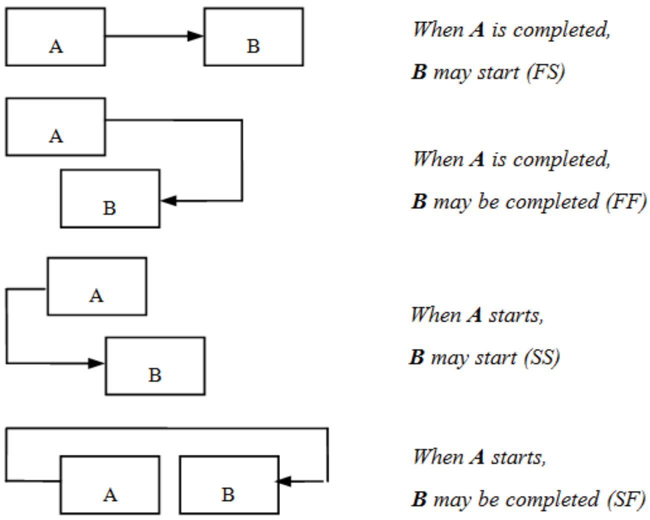

Fig. 1. Possible relations between activities. (*Source*: Wysocki, McGary 2003)

The assumptions adopted in the novel scenario-based DPG rule (SB-DPG rule) are as follows:

- Four types of activity sets my occur in the network: 1) set of deterministic activities without scenarios DET(A), 2) set of deterministic activities with scenarios DET(A)S, 3) set of alternative activities without scenarios ALT(A), 4) set of alternative activities with scenarios ALT(A)S. All tasks belonging to DET(A) and DET(A)S have to be executed. In the case of ALT(A) and ALT(A)S at least one activity from each set has to be performed;
- The number of scenarios for particular tasks may be different. Possible durations are estimated by experts;
- Four types of time dependencies are possible (Fig. 1): Finish to Start (FS), Finish to Finish (FF), Start to Start (SS) and Start to Finish (SF), see PDM (ang. *Precedence Diagramming Method*);
- The network is presented on the basis of AOA;
- Activities are executed without interruptions;
- The decision maker (project manager) has to declare his/her coefficient of optimism (β) for each activity belonging to sets DET(A)S and ALT(A)S (the level of that parameter may be the same for the whole project). The coefficient satisfies the following conditions: $\alpha, \beta \in [0,1]$ and $\alpha + \beta = 1$ where α denotes the coefficient of pessimism (α is close to 1 for extreme pessimists – risk averse behaviour, and β is close to 1 for radical optimists – risk prone behaviour);
- For activities with scenarios (see sets DET(A)S and ALT(A)S) a weighted average is calculated on the basis of a hybrid of Hurwicz and Bayes rules (H+B rule) which is described in detail in (Gaspars-Wieloch 2014a, 2015a, 2015b, 2016a, 2016b). In H+B rule, in contradiction to well-known classical decision rules such as Hurwicz, Wald, Hayashi, Savage approaches (Hayashi 2008; Hurwicz 1952; Savage 1961; Wald

1950), all outcomes have an influence on the value of the final measure, which is quite advantageous for cases where alternatives contain many scenario values almost equal to extreme ones. Weights used in the final measure depend on the DM's nature. The general idea of H+B is to assign, for a pessimist, α to the last term of the non-increasing sequence of all values related to a given decision and β to the remaining terms of that sequence. For an optimist, weights are set in a different way: β is connected with the first term of the sequence and α with the remaining ones. Such an assignment of parameters α and β to particular scenario values, depending on the level of optimism is justified in (Gaspars-Wieloch 2014a) where the author suggests a significant modification of the classical Hurwicz decision rule and adds to that procedure some features characteristic of Bayes rule. According to the investigation described in (Gaspars-Wieloch 2014b) the original version of the Hurwicz rule leads sometimes to illogical choices because it does not take into consideration the dispersion of payoffs connected with a given decision. The idea of the hybrid is to recommend for a strong pessimist an alternative with a relatively high payoff $a_{j,min}$ or with quite frequent payoffs (almost) equal to $a_{j,max}$. On the other hand, that rule suggests for an strong optimist an alternative with the highest (or almost the highest) payoff $a_{j,max}$, but its highest payoffs do not have to be frequent (since the optimist counts on luck). Of course, in the context of project network optimization, values aforementioned may represent for instance times, costs, resources (minimized criteria) or quality levels (maximized criterion) and decisions signify alternative tasks;

– The goal is to minimize project costs. Times are uncertain for some tasks and their costs partially depend on activity durations;

– The obtained optimal solution is used merely for one project since after the execution of that project some parameters in a new similar project may change (scenarios, times, costs, network structure, attitude towards risk etc.).

The novel scenario-based DPG rule (SB-DPG) for innovative projects contain the following steps:

1) Present the project network structure and define all parameters concerning particular activities (activity type, scenario time values, fixed and variable costs, time dependencies between tasks);
2) Declare the coefficient of optimism common for the whole project or separate for each task (group of tasks);
3) Solve the optimization model (12)–(25);
4) If any changes concerning activity parameters occur before (proactive management) or during (reactive management) the project execution, include those modifications in the optimization model and solve it.

$$C_u t_n + \sum_{<i,j>\in DET(A)} \left(c_{ij} + c_{ij}^u t_{ij}\right) + \sum_{<i,j>\in DET(A)S} \left(c_{ij} + c_{ij}^u t_{ij}^{hb}\right) + \\ + \sum_{<i,j>\in ALT(A)} \left(c_{ij} + c_{ij}^u t_{ij}\right) x_{ij} + \sum_{<i,j>\in ALT(A)S} \left(c_{ij} + c_{ij}^u t_{ij}^{hb}\right) x_{ij} \to \min \tag{12}$$

$$t_1 = 0 \tag{13}$$

$$t_j \geq t_i + t_{ij} \qquad <i,j>\in DET(A) \tag{14}$$

$$t_j \geq t_i + t_{ij}^{hb} \qquad <i,j>\in DET(A)S \tag{15}$$

$$t_j \geq (t_i + t_{ij}) \cdot x_{ij} \qquad <i,j>\in ALT(A) \tag{16}$$

$$t_j \geq (t_i + t_{ij}^{hb}) \cdot x_{ij} \qquad <i,j>\in ALT(A)S \tag{17}$$

$$\sum_{<i,j>\in S_i} x_{ij} = 1 \qquad i \in ALT(N) \tag{18}$$

$$t_j \geq t_i \qquad <i,j>\in DUM(A) \tag{19}$$

$$t_i, t_j \geq 0 \qquad i,j \in ALT(N), DET(N), END(N) \tag{20}$$

$$x_{ij} \in \{0,1\} \qquad <i,j>\in ALT(A) \tag{21}$$

$$t_{ij}^{hb(p)} = \frac{\alpha_{ij}^p \cdot t_{ij}^{z(ij)} + \beta_{ij}^p \cdot \sum_{s=1}^{z(ij)-1} t_{ij}^s}{\alpha_{ij}^p + (z(ij)-1) \cdot \beta_{ij}^p} \tag{22}$$

$$t_{ij}^{hb(o)} = \frac{\alpha_{ij}^{o} \cdot \sum_{s=2}^{z(ij)} t_{ij}^{s} + \beta_{ij}^{o} \cdot t_{ij}^{1}}{(z(ij)-1) \cdot \alpha_{ij}^{o} + \beta_{ij}^{o}} \tag{23}$$

$$t_{ij}^{0.5} = t_{ij}^{hb(p)} = t_{ij}^{hb(o)} = \frac{1}{z(ij)} \cdot \sum_{s=1}^{z(ij)} t_{ij}^{s} \tag{24}$$

$$t_{ij}^{hb} = \begin{cases} t_{ij}^{hb(p)}, & if \quad \beta_{ij} < 0.5 \\ t_{ij}^{hb(o)}, & if \quad \beta_{ij} > 0.5 \\ t_{ij}^{0.5}, & if \quad \beta_{ij} = 0.5 \end{cases} \tag{25}$$

where: C_u – unit project cost (parameter) – it's a cost dependent on the time of the whole project (not of particular activities), e.g. insurance, management, taxes; rent; t_n – project completion time (continuous variable); c_{ij} – fixed activity cost (parameter independent on activity duration); c^u_{ij} – unit activity cost (parameter dependent on activity duration); t_{ij} – time of activity <*i,j*> (parameter); t^{hb}_{ij} – weighted time of activity <*i,j*>; x_{ij} – binary variable connected with alternative activity <*i,j*> (it is equal to 1, when this task is done; otherwise it is equal to 0); t_i, t_j – times of event *i* and *j* (continuous variables); S_i – number of alternative activities belonging to job set *i* (parameter); $t^{hb(p)}_{ij}$ – pessimistic weighted time of activity <*i,j*> (parameter); $t^{hb(o)}_{ij}$ – optimistic weighted time of activity <*i,j*> (parameter); $t^{0.5}_{ij}$ – moderate time of activity <*i,j*> (parameter); α, β – coefficients of pessimism and optimism (parameters); $z(ij)$ – number of the last term in the non-decreasing sequence of scenario times for activity <*i,j*> – parameter (note that in this model, in contradiction to H+B rule, non-decreasing sequences are used since the time criterion is minimized); t^s_{ij} – number of the term in the non-decreasing sequence of scenario times for activity <*i,j*> (parameter).

As we can see, the decision rule is assisted with an optimization model containing continuous and binary variables. The model does not take into account time dependencies presented in Figure 1 since each type of time dependency must be considered individually in the form of additional constraints. Nevertheless, such an opportunity exists when scheduling a given project. The optimization model can be easily solved with the use of such computer tools as SAS/OR, MiniZinc, CPLEX or "R". The denominators in Eqns (22)–(24) are used in order to obtain weighted times not shorter than the shortest one (i.e. t^1_{ij}) and not longer than the longest one (i.e. $t^{z(ij)}_{ij}$). Of course, Eqn (18) applied in the model may be replaced by Eqns (9), (10) or (11), if necessary. The main advantages of SB-DPG rule are as follows. Firstly, it gives the possibility to consider more than three states of nature (in contradiction to PERT). Secondly, it allows inserting a different number of events for particular tasks. Thirdly, it does not require the probability estimation – it only uses probability-like quantities on the basis of the DM's attitude towards risk measured by the coefficient of optimism. Fourthly, it enables analyzing four categories of activities (depending on the deterministic or alternative structure and on deterministic or scenario times). Fifthly, it gives the opportunity to include time dependencies. Sixthly, it is attractive for passive project managers since only the coefficient of optimism (which can be common for all activities with scenarios) is required in the most simplified version of the model.

Case study

Now, let us illustrate the procedure described in the previous Section. Data related to a fictitious project are gathered in Table 1. Figure 2 presents its structure. The unit project cost is equal to 3 thousands euros per day. Scenario times for particular tasks are asymmetric. In the majority of cases, the relation between activities is described by means of *FS*, which signifies that when one task is completed, the second one can start. Additionally, we may notice that for two jobs other types of time dependency must be satisfied: D may finish at least one day after the beginning of G (*SF(1)*) and the duration of activity F is equal to the half of the weighted time of task G – step 1.

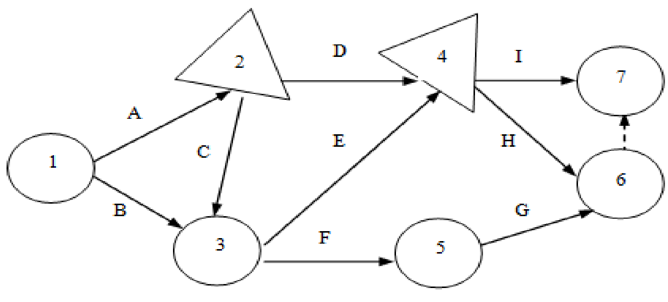

Fig. 2. Decision network project (example) (*Source*: prepared by the author)

Table 1. Activity parameters (*Source*: prepared by the author)

Activities	Set	Time (in days)	Time dependency	Fixed cost (in thousands Euros)	Unit cost (in thousands Euros)
A	DET(A)	5	AD – FS, AC – FS	4.0	0.5
B	DET(A)S	Scenarios: 3; 6; 8; 9; 15	BE – FS, BF – FS	3.0	0.2
C	ALT(A)S	Scenarios: 7; 10; 6; 3	CE – FS, CF – FS	5.0	0.7
D	ALT(A)	5	DI – FS, DH – FS	2.0	1.0
E	DET(A)	6	EI – FS, EH – FS	6.0	1.5
F	DET(A)	0.5·thb56	FG – FS	4.0	1.0
G	DET(A)S	Scenarios: 11; 17; 15	GD – SF(1)	3.0	0.4
H	ALT(A)S	Scenarios: 12; 20; 22; 15	–	2.0	0.6
I	ALT(A)S	Scenarios: 8; 10; 25	–	5.0	0.5
D1	DUM(A)	0	–	–	–

We assume that the project manager is a moderate optimist and declares a coefficient of optimism equal to 0.7 for activities B and C, and equal to 0.6 for jobs G, H and I (step 2). In step 3 we solve the optimization model:

$$3\cdot t_7+\left(6.5+15+(4+1\cdot t_{35})\right)+\left((3+7.11\cdot 0.2)+(3+13.86\cdot 0.4)\right)$$
$$+\left(7x_{24}\right)+\left((5+5.63\cdot 0.7)x_{23}+\left(2+16.67\cdot 0.6\right)x_{46}+\left(5+13.43\cdot 0.5\right)x_{47}\right)\rightarrow \min \tag{26}$$

$$t_1=0 \tag{27}$$

$$t_2\geq t_1+5; t_4\geq t_3+6; t_5\geq t_3+t_{35} \tag{28}$$

$$t_3\geq t_1+7.11; t_6\geq t_5+13.86 \tag{29}$$

$$t_4\geq (t_2+5)\cdot x_{24} \tag{30}$$

$$t_3\geq (t_2+5.63)\cdot x_{23}; t_6\geq (t_4+16.67)\cdot x_{46}; t_7\geq (t_4+13.43)\cdot x_{47} \tag{31}$$

$$x_{23}+x_{24}=1; x_{46}+x_{47}=1 \tag{32}$$

$$t_7\geq t_6 \tag{33}$$

$$t_1,t_2,t_3,t_4,t_5,t_6,t_7\geq 0 \tag{34}$$

$$x_{23},x_{24},x_{46},x_{47}\in\{0,1\} \quad (35$$

$$t_{13}^{hb(o)}=\frac{0.3\cdot(6+8+9+15)+0.7\cdot 3}{(5-1)\cdot 0.3+0.7}=7.11; t_{23}^{hb(o)}=5.63; t_{56}^{hb(o)}=$$
$$13.86; t_{46}^{hb(o)}=16.67; t_{47}^{hb(o)}=13.43 \tag{36}$$

$$t_{35}=0.5\cdot t_{56}^{hb(o)}=0.5\cdot 13.86=6.93 \tag{37}$$

$$t_4\geq t_5+1;\qquad D\rightarrow ALAP;\qquad G\rightarrow ASAP \tag{38}$$

Formulas (37) and (38) are added to consider two additional time dependency assumptions. The model has been solved by means of SAS/OR. The solution is: x_{23}=0; x_{24}=1; x_{46}=0; x_{47}=1; t_1=0; t_2=5; t_3=7.11; t_4=15.04; t_5=14.04; t_6=28.47; t_7=28.47; objective function = 149.53. Hence, the project manager should select alternative activities D and I. The minimum weighted total cost is equal to 149.53 thousands euros. The weighted project completion time equals 28.47 days. In connection with constraint GD – *SF(1)*, task D must finish as late as possible (i.e. at moment t^{II}_4) and task G must start as soon as possible (i.e. at moment t^{I}_5).

Discussion and conclusions

The paper presents a novel scenario-based decision project graph rule especially designed for innovative projects. The procedure allows taking into consideration uncertain information concerning activity times and costs. The number of scenarios for each task may be different. The approach successfully deals with networks with deterministic and alternative jobs. Uncertain parameters may be related to both of them, which gives the possibility to analyze problems taking place in a more practical environment. Due to the fact that innovative projects are characterized by many unknown items, the decision rule even does not require the probability estimation. Instead of it, probability-like quantities are generated on the basis of coefficients of optimism/pessimism declared by the decision maker (project manager). In the most simplified version of the optimization model those coefficients may have respectively common values for each activity, which is extremely attractive for passive project managers. The new method, similarly to GAAN (Golenko-Ginzburg, Blokh 1997), enables analyzing different kinds of decision makers (risk-neutral, risk-seeking, risk-aversion). In contradiction to many existing techniques, the novel approach gives the opportunity to additionally include different types of time dependencies concerning activity durations, beginnings and ends. The rule refers to Hurwicz and Bayes procedures – this combination is quite desirable especially in the case of asymmetric time distributions occurring in the project optimization problem. The aforementioned optimization model is mixed – it contains continuous and binary variables and linear and non-linear constraints. It may be easily solved thanks to some computer optimization tools, such as MiniZinc, "R", cplex or SAS/OR. Here, the last enumerated software has been used. It is very comfortable both for optimal solution searching and diverse simulations. Simulations may result from the necessity to modify selected project parameters (precedence relation, time dependency, possible scenarios, cost level, job sets etc.). Simulations support the reactive (during project execution) and proactive (before project execution) project management (Janczura, Kuchta 2011; Kuchta, Ślusarczyk 2015). As a matter of fact, the procedure proposed in the paper may be applied not only to totally new projects, but also to any other project in the case where the project manager does not want to make use of historical data or expects some significant changes in comparison to previous similar projects.

It is worth emphasizing that in the suggested optimization model the minimized objective function concerns only direct and indirect costs. However, it can also involve other financial categories, such as penalties and rewards resulting from the difference between the project completion time and the due date.

In the future it would be desirable to formulate a scenario-based model considering limited renewable and non-renewable resources. We are also interested in investigating different uncertain cost cases since in the optimization model formulated in the contribution the cost uncertainty just results form uncertain durations. However, sometimes in real project scheduling problems uncertainty related to costs has to be considered in a different way.

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References

Anholcer, M.; Gaspars-Wieloch, H. 2011. Efficiency analysis of the Kaufmann and Desbazeille algorithm for the deadline problem, *Operations Research and Decisions* 21(1): 5–18.

Anholcer, M.; Gaspars-Wieloch, H. 2013. Accuracy of the Kaufmann and Desbazeille algorithm for time-Cost trade-off project problems, *Statistical Review* [Przegląd Statystyczny] 3: 341–358.

Azeem, S. A. A.; Hosny, H. E.; Ibrahim, A. H. 2014. Forecasting project schedule performance using probabilistic and deterministic models, *HBRC Journal* 10(1): 35–42. https://doi.org/10.1016/j.hbrcj.2013.09.002

Benati, S. 2006. An optimization model for stochastic project networks with cash flows, *Computational Management Sciences* 3(4): 271–284. https://doi.org/10.1007/s10287-006-0018-8

Birge, J. R.; Louveaux, F. 2011. Uncertainty and modeling issues. Charter 2, in J. R. Birge, F. Louveaux (Eds.). *Introduction to stochastic programming*. Springer Series in Operations Research and Financial Engineering, 55–100. https://doi.org/10.1007/978-1-4614-0237-4_2

Błaszczyk, P.; Błaszczyk, T.; Kania, M. B. 2013. Project scheduling with fuzzy cost and schedule buffers. Chapter *Lecture Notes in Electrical Engineering* 170: 375–388. https://doi.org/10.1007/978-94-007-4786-9_30

Broadleaf. 2014. *Beta PERT origins*. Creating value from uncertainty. Broadleaf Capital International Pty Ltd. [online], [cited 01 February 2017]. Available from Internet: http://broadleaf.com.au/wp-content/uploads/2014/07/Beta-PERT-origins-2014-v2.pdf

Creemers, S.; De Reyck, B.; Leus, R. 2013. *Project planning with alternative technologies in uncertain environments*. KU Leuven. Faculty of Economics and Business.

Creemers, S.; Leus, R.; Lambrecht, M. 2010. Scheduling Markovian PERT networks to maximize the net present value, *Operations Research Letters* 38(1): 51–56. https://doi.org/10.1016/j.orl.2009.10.006

Crowston, W. B. 1970. Decision CPM: Network reduction and solution, *Operational Research Quaterly* 21(4): 435–452. https://doi.org/10.1057/jors.1970.93

Crowston, W. B.; Thompson, G. L. 1967. Decision CPM: a method for simultaneous planning, scheduling and control of Project, *Operations Research* 15: 407–426. https://doi.org/10.1287/opre.15.3.407

De Reyck, B.; Leus, R. 2008. R&D-project scheduling when activities may fail, *IIE Transactions* 40(4): 367–384. https://doi.org/10.1080/07408170701413944

Dodin, B. 2006. A practical and accurate alternative to PERT. Chapter in *Perspectives in Modern Project Scheduling. International Series in Operations Research and Management Science* 92: 3–23.

Dubois, D.; Prade, H. 2012. Gradualness, uncertainty and bipolarity: making sense of fuzzy sets, *Fuzzy Sets and Systems* 192: 3–24. https://doi.org/10.1016/j.anucene.2015.08.002

Gaspars-Wieloch, H. 2008. *Methods of time-cost project optimization*: Doctoral Thesis. Poznań University of Economics.

Gaspars-Wieloch, H. 2012. Time-cost project management with Solver, *Contemporary Issues in Business, Management and Education'2012*. http://dx.doi.org/10.3846/cibme.2012.43

Gaspars-Wieloch, H. 2014a. Propozycja hybrydy reguł Hurwicza i Bayesa w podejmowaniu decyzji w warunkach niepewności [A hybrid of the Hurwicz and Bayes rules in decision making under uncertainty]. Charter in T. Trzaskalik (Ed.). *Modelowanie preferencji a ryzyko*'14. *Studia Ekonomiczne*. *Zeszyty Naukowe Uniwersytetu Ekonomicznego w Katowicach* 178: 74–92 (in Polish).

Gaspars-Wieloch, H. 2014b. Modifications of the Hurwicz's decision rules, *Central European Journal of Operational Research* 22(4): 779–794. https://doi.org/10.1007/s10100-013-0302-y

Gaspars-Wieloch, H. 2015a. Modifications of the Omega ratio in decision making under uncertainty, *Croatian Operational Research Review* 6(1): 181–194. https://doi.org/10.17535/crorr.2015.0015

Gaspars-Wieloch, H. 2015b. Innovative products and newsvendor problem under uncertainty without probabilities. Charter, in S. L. Zadnik, J. Zerownik, M. Kljajic Borstnar, S. Drobne (Eds.). *Proceedings of the 13th International Symposium of Operational Research* SOR'15. Slovenian Society INFORMATIKA (SDI). Section for Operational Research (SOR), 343–350.

Gaspars-Wieloch, H. 2015c. A decision rule supported by a forecasting stage based on the decision maker's coefficient of optimism, *Central European Journal of Operational Research* 23(3): 579–594. https://doi.org/10.1007/s10100-014-0364-5

Gaspars-Wieloch, H. 2016a. Resource allocation under complete uncertainty – case of asymmetric payoffs, *Organization and Management* [Organizacja i Zarzadzanie] 96: 247–258.

Gaspars-Wieloch, H. 2016b. Newsvendor problem under complete uncertainty: a case of innovative products, *Central European Journal of Operations Research*, 1–25. https://doi.org/10.1007/s10100-016-0458-3

Gaspars-Wieloch, H. 2017. A decision rule based on goal programming and one-stage models for uncertain multi-criteria mixed decision making and games against nature, *Croatian Operational Research Review* 8(1) (in print).

Golenko-Ginzburg, D. 1988. Controlled activity networks for project management, *European Journal of Operations Research* 37(3): 336–346. https://doi.org/10.1016/0377-2217(88)90196-8

Golenko-Ginzburg, D.; Blokh, D. 1997. A generalized activity network model, *Journal of the Operational Research Society* 48(4): 391–400. https://doi.org/10.1057/palgrave.jors.2600378

Grudzewski, W. 1985. *Badania operacyjne w organizacji i zarządzaniu* [Operations research in organization and management]. PWN. Warsaw (in Polish)

Guney, S.; Newell, B. R. 2015. Overcoming ambiguity aversion through experience, *Journal of Behavioral Decision Making* 28(2): 188–199. https://doi.org/10.1002/bdm.1840

Guo, P. 2011. One-shot decision theory, *IEEE Transactions on Systems, Man and Cybernetics*. Part A 41(5): 917–926. https://doi.org/10.1109/TSMCA.2010.2093891

Hastings, N. A. J.; Mello, J. M. C. 1978. *Decision networks*. Chichester, New York: Willey.

Hayashi, T. 2008. Regret aversion and opportunity dependence, *Journal of Economic Theory* 139(1): 242–268. https://doi.org/10.1016/j.jet.2007.07.001

Hindelang, T. J.; Muth J. F. 1979. A dynamic programming algorithm for Decision CPM networks, *Operations Research* 27(2): 225–241. https://doi.org/10.1287/opre.27.2.225

Hurwicz, L. 1952. *A criterion for decision making under uncertainty*. Technical Report 355. Cowles Commission.

Igelmund, G.; Radermacher, F. J. 1983. Preselective strategies for the optimization of stochastic project networks under resource constraints, *Networks* 13(1): 1–28. https://doi.org/10.1002/net.3230130102

Janczura, M.; Kuchta, D. 2011. Proactive and reactive scheduling in practice, *Research Papers of Wroclaw University of Economics* 238: 34–51.

Kaplan, S.; Barish, N. N. 1967. Decision-making allowing for uncertainty of future investment opportunities, *Management Science* 13(10): 569–577. https://doi.org/10.1287/mnsc.13.10.B569

Kmietowicz, Z. W.; Pearman, A. D. 1984. Decision theory, linear partial information and statistical dominance, *Omega* 12: 391–399. https://doi.org/10.1016/0305-0483(84)90075-6

Knight, F. H. 1921. *Risk. uncertainty. profit.* Hart. Boston MA. Schaffner & Marx. Houghton Mifflin Co.

Kuchta, D.; Ślusarczyk, A. 2015. Application of proactive and reactive project scheduling – case study, *Research Papers of Wroclaw University of Economics* 386: 99–111. https://doi.org/10.15611/pn.2015.386.07

Magruk, A. 2016. Uncertainty in the sphere of the industry 4.0 – potential areas to research, *Business, Management and Education* 14(2): 275–291. https://doi.org/10.3846/bme.2016.332

Malcolm, D. G.; Rosenboom, J. H.; Clark, C. E.; Fazar, W. 1959. Application of a technique for research and development program evaluation, *Operations Research* 7(5): 646–669. https://doi.org/10.1287/opre.7.5.646

Merigo, J. M. 2015. Decision-making under risk and uncertainty and its application in strategic management, *Journal of Business Economics and Management* 2015(1): 93–116. https://doi.org/10.3846/16111699.2012.661758

Moder, J. J.; Phillips, C. R. 1964. *Project Management with CPM and PERT.* New York: Reinhold Publishing Corporation.

Neumann, K. 1990. *Stochastic project networks: temporal analysis, scheduling, and cost minimization*. Berlin: Springer-Verlag. https://doi.org/10.1007/978-3-642-61515-3

Neumann, K.; Steinhardt, U. 1979. GERT network and the time-oriented valuation of projects, *Lecture Notes in Economics and Mathematical Systems* 172.

Okmen, O.; Oztas, A. 2014. Uncertainty evaluation with fuzzy schedule risk analysis model in activity networks of construction projects, *Journal of South African Institution of Civil Engineering* 56(2).

Pollack-Johnson, B.; Liberatore, M. J. 2005. Project planning under uncertainty using scenario analysis, *Project management Journal* 36(1): 15–26.

Pritsker, A. A. B. 1979. *Modeling and analysis using Q-GERT Networks*. 2nd ed. Wiley.

Pritsker, A. A. B.; Happ, W. W. 1966. GERT: Graphical evaluation and review technique – Part 1. Fundamentals, *Journal of Industrial Engineering* 17: 267–274.

Project Management Institute. 2000. *A guide to the project management body of knowledge (PMBOK® Guide)*. Newtown Square, Pennsylvania.

San Cristobal, M. J. R. 2015. *Management science, operations research and project management: modeling, evaluation, scheduling, monitoring*. Gower Applied Business Research.

Savage, L. 1961. *The foundations of statistics reconsidered*. Studies in Subjective Probability. New York: Wiley, 173–188.

Sobel, M. J.; Szmerekovsky, J. G.; Tilson, V. 2009. Scheduling projects with stochastic activity duration to maximize expected net present value, *European Journal of Operational Research* 198(1): 697–705. https://doi.org/10.1016/j.ejor.2008.10.004

Spinner, M. 1981. *Elements of project management: plan, schedule and control.* Prentice-Hall, Englewood Cliffs, New Jersey.

Suhobokov, A. 2007. Application of Monte Carlo simulation methods in risk management, *Journal of Business Economics and Management* 8(3): 165–168.

Thompson, G. L. 1968. CPM and DCPM under risk, *Naval Research Logistics* 15(2): 233–239. https://doi.org/10.1002/nav.3800150208

Trzaskalik, T. 2008. *Wprowadzenie do badan operacyjnych z komputerem* [Introduction to operations research with computer]. 2nd ed. Warsaw: Polskie Wydawnictwo Ekonomiczne (in Polish).

von Mises, L. 1949. *Human action. A treatise on economics*. The Ludwig von Mises Institute. Auburn. Alabama.

von Neumann, J.; Morgenstern, O. 1944. *Theory of games and economic behavior.* New York. Princeton: Princeton University Press.

Voronova, I. 2008. Methods of analysis and estimation of risks at the enterprises of non-financial sphere at Latvia, *Journal of Business Economics and Management* 9(4): 319–326. https://doi.org/10.3846/1611-1699.2008.9.319-326

Voropayev, V. I.; Gelrud, Y. D.; Golenko-Ginzburg, D. 2013. Decision making in controlled cyclic alternative network projects with deterministic branching outcomes, *PM World Journal* II(IX).

Wald, A. 1950. *Statistical decision functions*. New York: Wiley.

Ward, S.; Chapman, C. 2003. Transforming project risk management into project uncertainty management, *International Journal of Project Management* 21: 97–105. https://doi.org/10.1016/S0263-7863(01)00080-1

Wiest, J. D.; Levy, F. K. 1977. *A management guide to PERT/CPM with GERT/PDM/DCPM and other networks.* 2nd ed. Englewood Cliffs, NJ: Prentice-Hall.

Wysocki, R. K.; McGary, R. 2003. *Effective project management: traditional, adaptive, extreme*. 3rd ed. Wiley.

Zhang, W.; Padmanabhan, P.; Huang, C.-H. 2015. Sequential capital investment decision making under extreme cash flow situations: evidence using Monte Carlo simulation, *Journal of Business Economics and Management* 16(5): 877–900. https://doi.org/10.3846/16111699.2015.1039056